**Normal Distributions**

* **Normal (Gaussian) distribution**
  + R. v. X has a normal distribution with parameters μ and σ − i.e. X ~ N(μ, σ) − if its PDF is given by
    - f(x) = f(x, μ, σ) =
    - Where μ = mean and σ = standard deviation
    - Notation can also be X ~ N(μ, σ2) – mean & variance as parameters
    - E(X) = μ
    - Var(X) = σ2
  + If X ~ N(μ1, σ1) and Y ~ N(μ2, σ2)
    - Then X + Y ~ N(μ1 + μ2, √(σ12 + σ22))
    - Any linear combination: aX + bY + c ~ N(aμ1 + bμ2 + c, √(a2σ12 + b2σ22)
  + The PDF f(x) is not integrable, hence the normal distribution table is used
  + **Standard normal distribution**
    - If X ~ N(μ, σ), Z = (X − μ)/σ ~ N(0, 1)
    - Because the PDF is symmetric, P(Z > z) = P(Z < -z) for any z ∈ R
  + Ex: bolts produced have diameters Y ~ N(10, 1)
    - Washers produced have inner diameters X ~ (11, 0.5)
    - P(bolt fits inside washer) = P(X > Y) = P(X – Y > 0) = ?
    - X – Y ~ N(11 – 10, √(0.52 + (-1)212)) = N(1, 1.118)
    - Z = (X – Y – 1)/1.118 ~ N(0, 1)
    - P(X – Y > 0) = P((X – Y – 1)/1.118 > (0 – 1)/1.118))

= P(Z > -0.89)

= P(Z < 0.89) by symmetry

= 0.8133 from table

* **Central Limit Theorem**
  + Consider r. v. X1, X2, … Xn and Sn = X1 + … Xn, Xn-bar = (X1 + … Xn)/n
  + If Xi ~ N(μ, σ) (independent) for all i = 1 … n then Sn ~ N(nμ, σ√n)
    - E(Sn) = E(X1 + … + Xn) = μ + … + μ = nμ
    - Var(Sn) = Var(Xi + … + Xn) = σ2 + … σ2 = nσ2
  + And Xn-bar ~ N(μ, σ/√n) (exact distribution)
  + Let X1 … Xn be independent identically distributed r. v. (can be different distr. but same type)
  + Then Sn ~ N(nμ, σ√n) (approx.) and Xn-bar ~ N(μ, σ/√n) (approx.) as n → ∞
    - In general n ≥ 30